June 2018

To Advanced Placement Calculus Student,

I congratulate you on accepting the challenge of taking the Advanced Placement Calculus course at Northwestern. I have prepared this packet to give you some information about the course and help you get ready for Calculus.

The Advanced Placement Calculus course offers secondary school students the opportunity to pursue and receive credit for college-level course work at the secondary level. The curriculum that I will follow this year is the one that is recommended by The College Board. Their curriculum is based on the premise that college-level material can be taught successfully to able and well-prepared secondary school students. We will cover all of the topics in the Calculus AB curriculum which corresponds to *at least* as much material as a standard first semester course of college Calculus.

One of the focal points of the course is the preparation for the 3 ¼ hour Advanced Placement Calculus AB Examination given in May 2019. This is a national exam that you will be expected to take as one of the requirements of this course. There is a registration fee required of about $94.00 that will be your responsibility. Please see me if this fee presents a problem for you. The AP Exam is graded on a scale of one to five and the results are used by many colleges and universities for placement purposes. It is possible for you to gain college credit and/or advanced placement as a result of your grade on this test. The Advanced Placement Exam requires the use of a graphing calculator. Students have found it helpful to own a TI -89, but you can get by using the TI-83/84.

AP Calculus is a rigorous and demanding course. You are among the best math students in the country. You should be proud of that fact and be willing to develop your talents to their fullest. My goal is to help you accomplish that development, but I can only be a guide. It is up to you to put forth the kind of consistent effort necessary to achieve your potential.

I have included a copy of the course overview in this packet. I expect that by the first day of school you will have familiarized yourself with the overview and that you will bring to school on that first day your organized notebook. I have also included a homework assignment that is due on the first day of school.

Have a wonderful and relaxing summer. I am looking forward to delving into the study and exploration of a branch of mathematics which my college Calculus book refers to as "one of the supreme accomplishments of the human intellect." Calculus truly is a fascinating course, and together we will appreciate its complexities, eloquence, and problem solving capabilities.

Sincerely,

Mrs. Jakobsen

A. If you purchased a TI-89 calculator, you can refer to the tutorial at http://www.prenhall.com/divisions/esm/app/graphing/ti89/. Begin using your new calculator as you complete the summer assignment so you will start to get familiar with the TI-89.

B. Set up an account on Khan Academy. We will use this site for part of the summer assignment and for several assignments throughout the school year. Go to khanacademy.org/coaches and type in:

YRTGXE72

C. Sign into to your Khan Academy account and watch the following videos (type the video name in the search bar).

1. Newton, Leibniz, and Usain Bolt
2. Derivative as slope of curve
3. Formal definition of the derivative as a limit

Take notes and respond to the questions listed for each video.

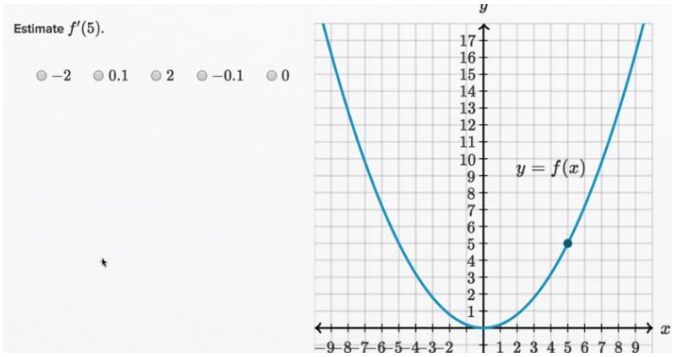
1. Newton, Leibniz, and Usain Bolt
   a. Sketch the graph of distance vs time.

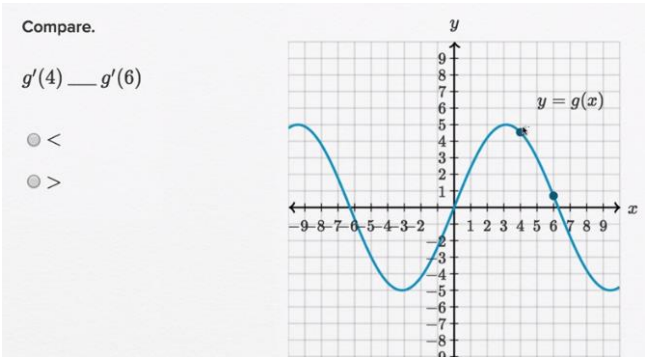
   b. What is Usain Bolt's average speed? Show how it is calculated.

   c. What is Usain Bolt's fastest instantaneous speed? Show the notation for this.

   d. How is the instantaneous rate of change related to the average rate of change?

2. **Derivative as slope of curve**





3. **Formal definition of the derivative as a limit**

   a. What algebra 1 formula is the derivative based on?

   b. How do we calculate the slope of the secant line? Draw the diagram that illustrates this.

   c. Write the formal definition of the derivative.

   d. Why does a secant line need to be involved in the definition of the derivative? Why can't we simply calculate the slope at a single point?

D. On page 100 in your calculus textbook, find the definition of the derivative at a point. Draw the diagram to support this definition. Write the definition.

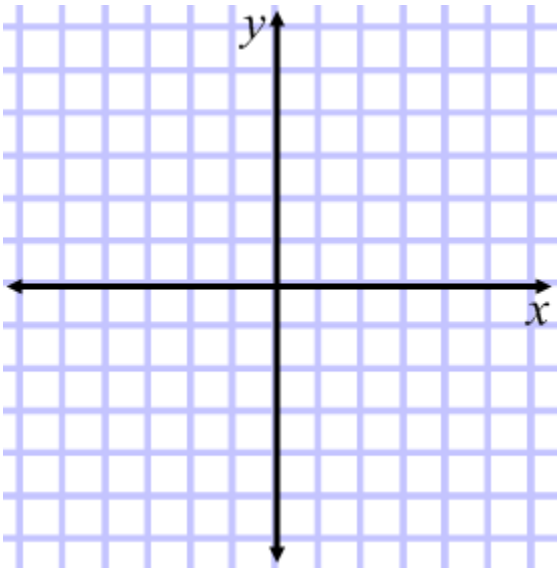
E. Complete the problems on the following pages. The work will be graded for correctness, not just effort. The following resources may be helpful.
   a. Your calculus textbook – chapters 1, 2 and section 3.1. You can also use the index to look up a topic.
   b. Khan Academy

1. Graph each function, clearly marking at least 3 points on each graph. List the domain and range.

a. $f(x) = -\sqrt[3]{x} + 2$

Domain:

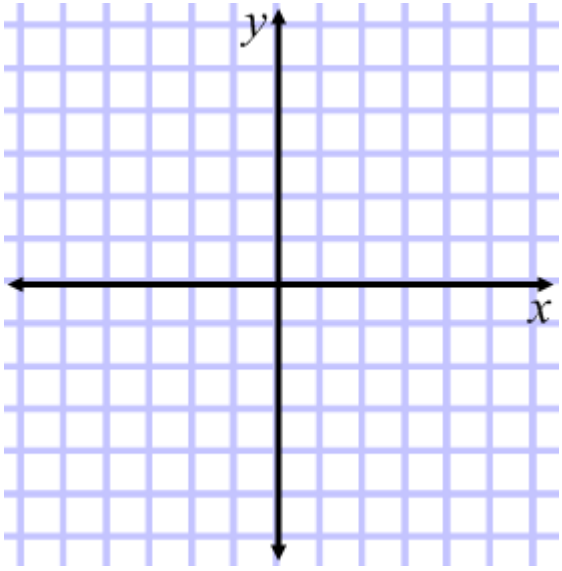
Range:



b. $f(x) = 2\sqrt{-(x-3)}$

Domain:

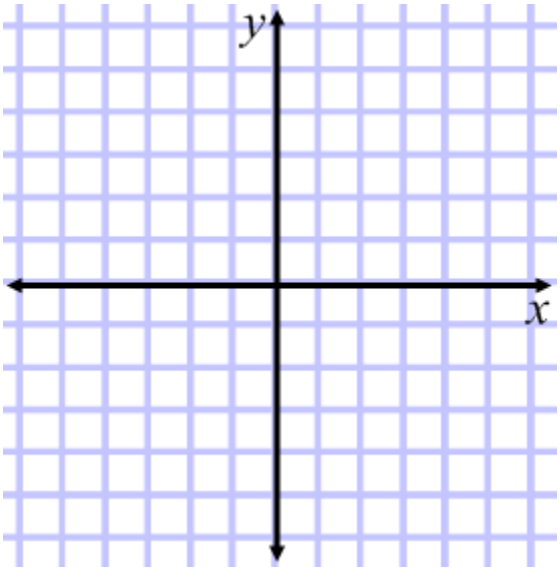
Range:



c. $f(x) = \frac{1}{x+2} - 3$

Domain:

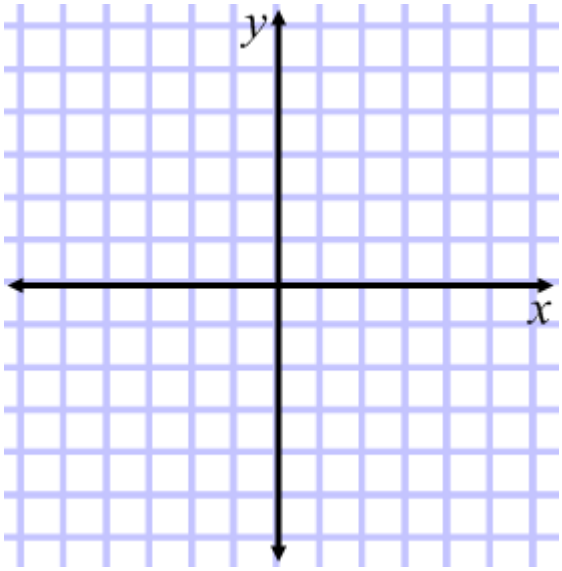
Range:



d. $f(x) = -\frac{1}{2}(x-1)^2 + 3$

Domain:

Range:

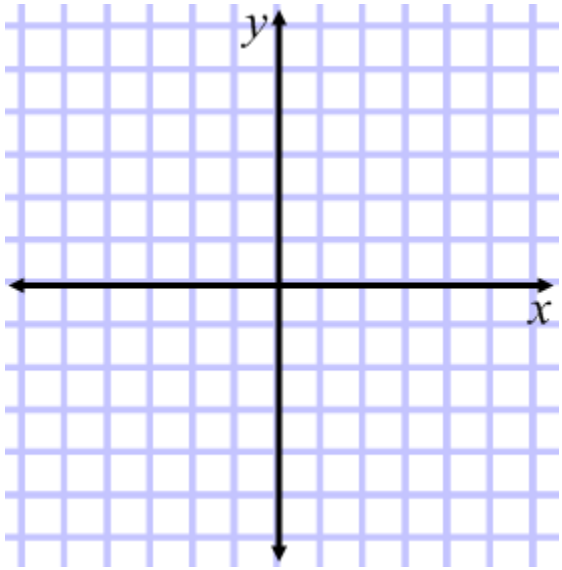


Graph each function, clearly marking at least 3 points on each graph. List the domain and range.

e. $f(x) = \frac{2}{3}|x - 2| - 4$

Domain:

Range:

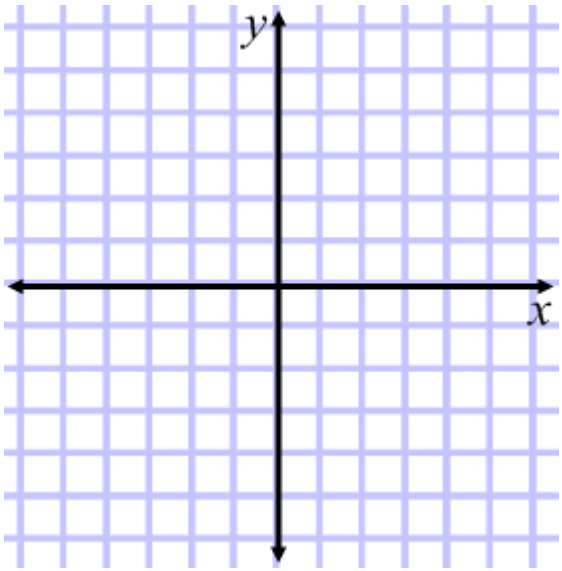


f. $f(x) = -3\cos\left(x + \frac{\pi}{4}\right) - 1$

Graph at least one full period. Label the x-axis in terms of π. Clearly mark the central axis, maxima, and minima.

Domain:

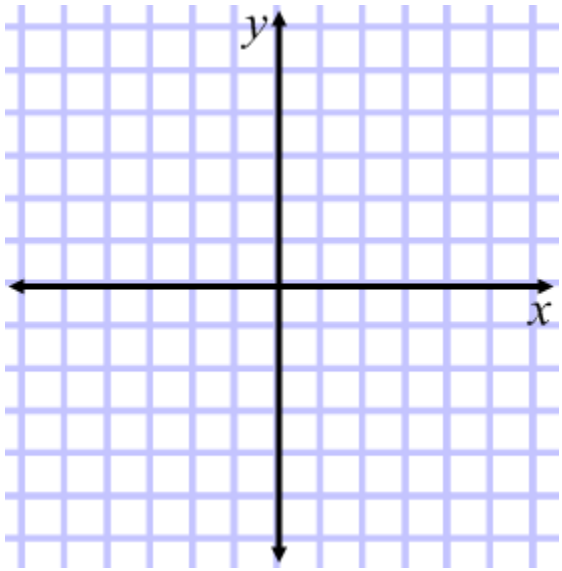
Range:



g. $f(x) = 2(0.5)^x - 4$

Domain:

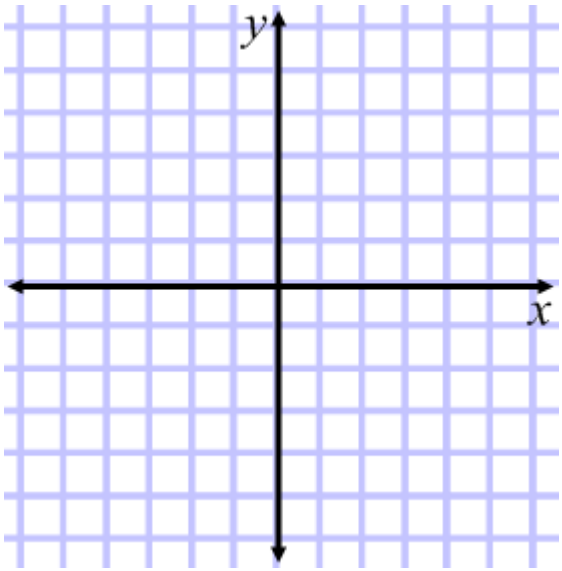
Range:



h. $f(x) = \log_2(x + 3)$

Domain:

Range:



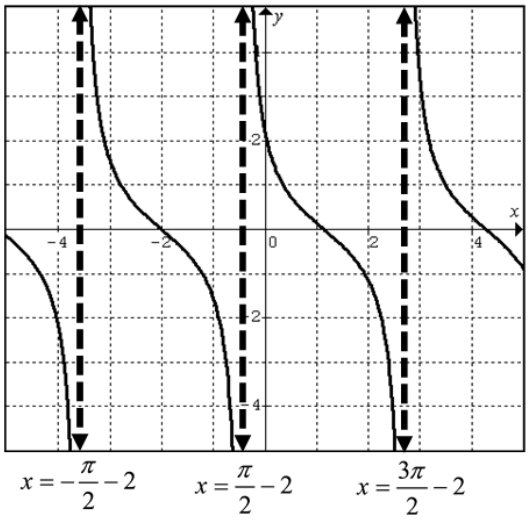
2. Write the equation for each graph. Also write the domain and range for each.

a.

Equation:

Domain:

Range:



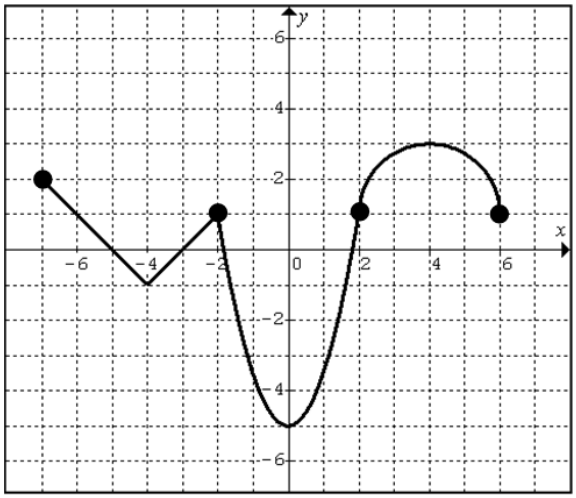
$x = -\frac{\pi}{2} - 2$ $x = \frac{\pi}{2} - 2$ $x = \frac{3\pi}{2} - 2$

b.

Equation:

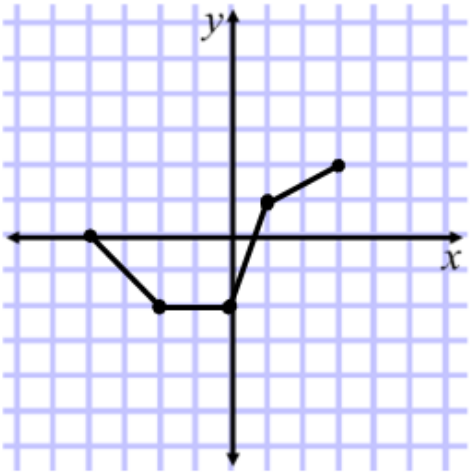
Domain:

Range:

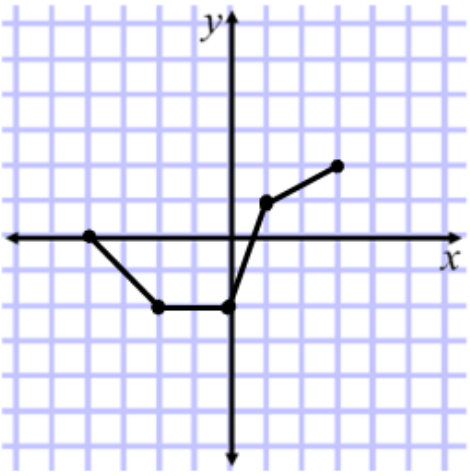


3. Use the graph of $y = f(x)$ below. Draw the graph using a colored pencil so it is easy to see.

a. Construct the graph of $g(x) = -3f(x-2)$.



b. Construct the graph of $h(x) = f^{-1}(x)$



4. State whether the function is even, odd, or neither. Show the use of the even/odd function tests to support your answer.
   a. $f(x) = \cos(x) - x^4$

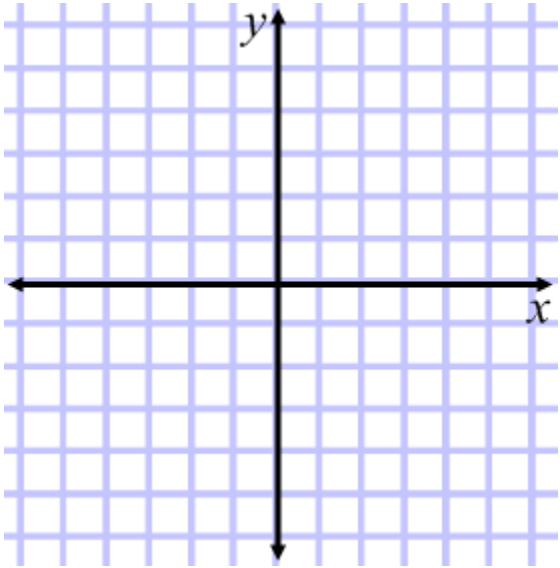
   b. $g(x) = \frac{|x|}{4-x^2}$

   c. $h(x) = 3x^3 - \frac{4}{x}$

5. Consider the function $f(x) = .5\sqrt{x+3}$ on the domain x ≥ -3.
   a. Find the inverse of $f(x)$.

   b. Graph both $f(x)$and $f^{-1}(x)$.

   c. Graph and write the equation of the line of symmetry between $f(x)$and $f^{-1}(x)$.



6. Find the inverse of $f(x) = \frac{x+1}{x-4}$.

7. Write two different sets of functions for $f(x)$ and $g(x)$ such that $(f \circ g)(x) = \frac{4}{\sqrt[3]{x+2}}$.

Set 1:	Set 2:
$f(x)$ =	$f(x)$ =
$g(x)$ =	$g(x)$ =

8. Use the composite function test to determine whether or not $f(x) = x^3 - 8$ and $g(x) = \sqrt[3]{x+2}$ are inverses of each other. Show both tests.

9. Graph the function on your calculator and state the intervals on which the function is increasing.

$$y = 2 + |x - 1|$$

10. Use your graphing calculator to find all relative maximum and relative minimum values of each function. Also state value of x at which each relative extremum occurs.

$$y = x^3 - 2x^2 + 3x - 1$$

11. Solve the equation algebraically. $\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2-4x+3}$

12. The Paniati Paper Company has been contracted to manufacture a box with no top that is to be made by removing squares of width x from the corners of a 10 inch by 15 inch piece of cardboard.
    a. Draw and label a diagram for the problem.

    b. Write an equation that models the volume of the box. Specify the domain for your equation.

    c. Determine x so that the box has a volume of 120 in^3.

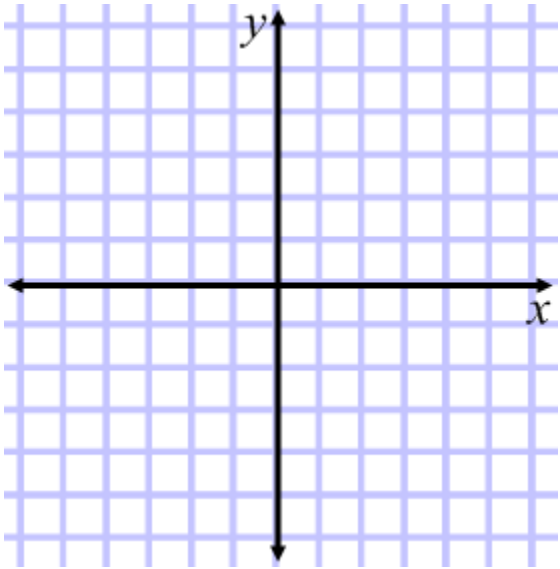
    d. What are the dimensions of the box that yield the maximum volume?

13. Consider the function $f(x) = \frac{x+3}{x^2+x-6}$.

    a. Graph the function.

    b. Identify the coordinates of any points of discontinuity.

    c. Write the equation of the horizontal asymptote. Use limits to represent the corresponding behavior.



    d. Write the equation of the vertical asymptote. Use limits to represent the corresponding behavior.

14. Write an exponential function of the form $y = Ae^{kt}$ that passes through the points (0,5) and (2,12). Show your work.

15. You invest $1,000 at 3.25% APR, compounded monthly. How many years will it take for the investment to grow to $2,000? Answer algebraically.

16. You invest $800 and interest is compounded continuously. If your money doubled in 12 years, what interest rate did you receive? Answer algebraically.

17. Solve each equation algebraically.
    a. $\log x^2 = 2$

    b. $\log(x) - \frac{1}{2}\log(x+4) = 1$

    c. $\ln(3x-2) + \ln(x-1) = 2\ln(x)$

18. Find the exact value of each. Do not use a calculator. Provide a sketch for each problem to support your answer.

a. $\sin 60°$

b. $\cos 150°$

c. $\tan(-225°)$

d. $\sec(135°)$

e. $\sin\frac{5\pi}{3}$

f. $\csc\frac{5\pi}{6}$

g. $\tan\left(-\frac{7\pi}{6}\right)$

h. $\sec 270°$

i. $\cot(-180°)$

j. $\tan 450°$

19. Find the exact value of all 6 trigonometric functions for the angle. Do not use your calculator.

    a. $\frac{19\pi}{3}$

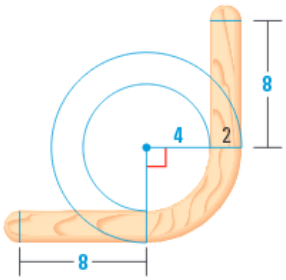
    b. $675°$

20. Determine the exact value of the other 5 trigonometric functions of Θ given that $\tan\theta = \frac{24}{7}$ and Θ is in quadrant I. Do not use your calculator.

21. Find the length of the arc intercepted by the central angle $\frac{2\pi}{5}$ in a circle with radius 4.

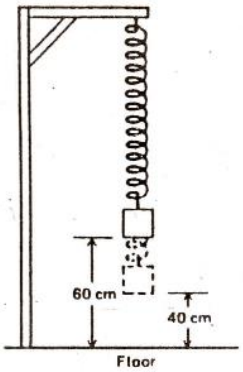
22. From the top of a 300-ft building, George notices a car moving towards him. If the angle of depression of the car changes from 15° to 23° during the observation, how far does the car travel? Draw a diagram and show all steps in solving the problem.

23. Find the area of the boomerang shown. The dimensions are given in inches. Give your answer in terms of π and to two decimal places.



24. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 seconds.

    a. Write an equation to represent the height if the weight as a function of time.



    b. Find the second time the weight reaches a height of 45cm. Answer algebraically, showing all your steps.

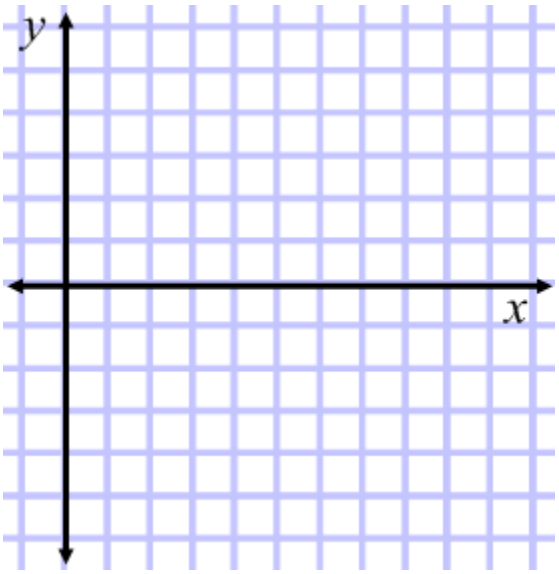
25. Simplify the expression to a single term. $\sin x(\tan x + \cot x)$

26. Prove the identity. $\tan^2\theta - \sin^2\theta = \tan^2\theta \cdot \sin^2\theta$

27. Find the (x,y) pair for the value of the parameter:
$x = t^2 + 3t$ and $y = \sqrt{t+3}$ for t = 1.

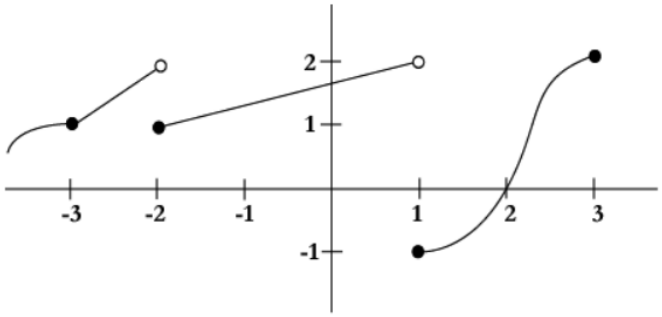
28. Graph the parametric equation. Also write the equivalent rectangular equation.

$x = 2t - 4, y = 3t - 14;\ 3 \leq t \leq 6$



29. Brian hits a baseball straight toward a 20-ft high fence that is 400 ft from home plate. The ball is hit when it is 3 ft above the ground and leaves the bat at an angle of 30°with the horizontal. Find the initial velocity needed to clear the fence. Show your work. Hint: Projectile motion.

30. Shown is the graph of $y = f(x)$.



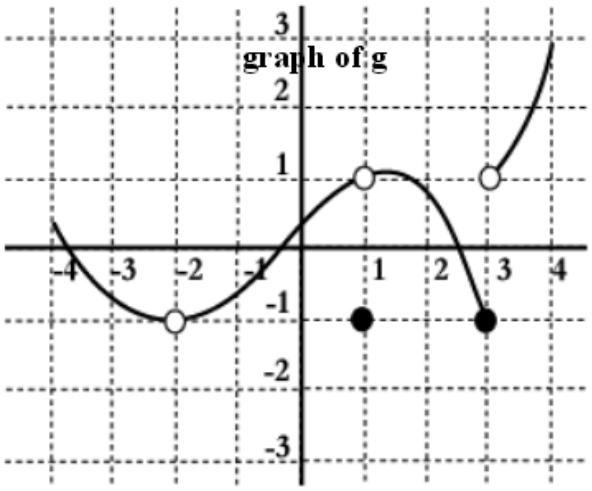
Evaluate the following:

a) $\lim_{x\to 1^-} f(x)$ b) $\lim_{x\to 1^+} f(x)$ c) $\lim_{x\to -2^+} f(x)$ d) $\lim_{x\to -2^-} f(x)$

e) $f(-2)$ f) $\lim_{x\to -2} f(x)$

g) Find the value(s) of a so that $\lim_{x\to a} f(x) = 1$

31. Let g be the function whose graph is shown below. Use the graph to evaluate each quantity if it exists. If it does not exist, explain why.



a) $\lim_{x\to -2} g(x)$ b) $\lim_{x\to 1} g(x)$ c) $g(1)$

d) $\lim_{x\to 3^-} g(x)$ e) $\lim_{x\to 3^+} g(x)$ f) $\lim_{x\to 3} g(x)$

32. Find each limit algebraically or explain how you determined its value (without a calculator). Show your work. You can use your calculator to verify your result.

a. $\lim_{x\to 3} \frac{x^2+9}{x-3}$

b. $\lim_{x\to 3} \frac{x^2-9}{x-3}$

c. $\lim_{x\to 1} \frac{x^2-1}{x^3-x^2}$

d. $\lim_{x\to \infty} \frac{3x-7}{2x+1}$

e. $\lim_{x\to \infty} \frac{1000x^2}{x^3-5}$

f. $\lim_{h\to 0} \frac{(2+h)^2-2^2}{h}$

g. $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$

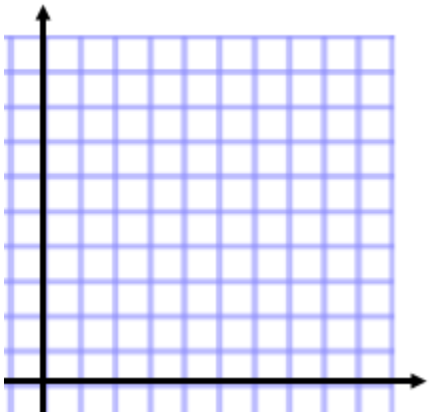
33. Consider the function $f(x) = 4x^2 - 3x + 2$.
    a. Use the definition of the derivative (at a point) to evaluate $f'(4)$. Show your work.

    b. Use the definition of the derivative (as a function) to evaluate $f'(x)$. Show your work.

    c. Substitute $x = 4$ into your answer to part b and verify that it matches your result from part a.

34. Suppose that the position of an object can be modeled by the function $x(t) = -0.4t^2 + 3t$, where t is measured in hours and x is measured in miles.
    a. Graph the function on the interval [0,7.5].

    b. Calculate the average rate of change of the object from t = 0 to t = 6. Show your work. Include units on your answer.



    c. Draw the secant line on the graph that corresponds to the average rate of change of the object from t = 0 to t = 6. Show/explain how this is connected.

    d. Calculate the instantaneous rate of change at t = 2.Include units on your answer.

    e. Draw the tangent line on the graph that corresponds to the instantaneous rate of change at t = 2. Show/explain how this is connected.